# First reduce a triangle…
(an exploration around the Thales property)

<u>Theorem</u> : If you cut a triangle, using a parallel to one side, so as to get a second triangle, then the two triangles have proportional sides.

This short presentation sets out to show how we can exploit this theorem in a research activity by analysing a diagram, proving the property which has been observed, generalizing, asking oneself new *questions*, changing the point of view by moving from the plane to the space, then asking oneself more new *questions*.

1- In the plane
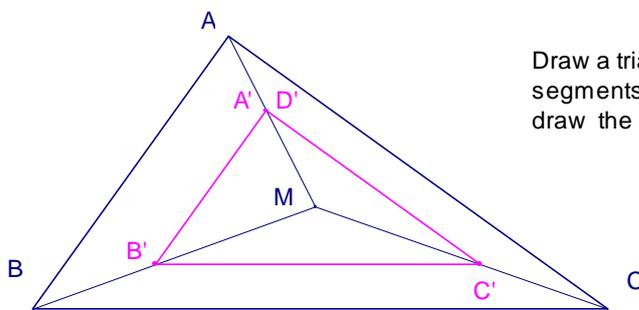
Draw a triangle ABC, choose a point M inside ABC, draw the segments [MA], [MB] and [MC] ; choose a point A' on [MA] and draw the parallels (A'B'), (B'C') and (C'D') to the sides of ABC.

We observe that D'=A', and to prove it we use the theorem three times in MAB, MBC and MCA (we get MA'=MD').
And *if we chose M outside the triangle (not on a side of ABC) ?* The proof is still valid.
Will this reduction method work with a quadrilateral ? a non-convex quadrilateral ? any polygon ? It does.

Cutting of along its sides three strips of different width has reduced ABC.
Is it possible to choose M in order to have strips of same width ? M must lie on the bisectors, thus must be the centre of the inscribed circle.
And with a convex quadrilateral ? a convex polygon ? M must also be the centre of the inscribed circle… if it exists !
How to recognize such a quadrilateral ? Draw one around a circle and it becomes obvious that the sums of the lengths of the opposite sides must be equal.
Note : the circumscribed circle exists if the sums of the angles of the opposite sectors are equal (to 180°).

We know the ratio of the sides of the two triangles A'B'C' and ABC : MA'/MA=k.
What is the ratio of their areas ? ABC is cut into three triangles with common vertex M ; using the theorem we prove that the ratio of the heights starting at M in MA'B' and MAB is also k ; we deduce that the ratio of their areas is k^2, and so for A'B'C' and ABC. This proof works fine for any convex polygon, and also for non-convex ones (with non crossed sides).

2- In the space

The first drawing shows a pyramid
(top view of a tetrahedron).
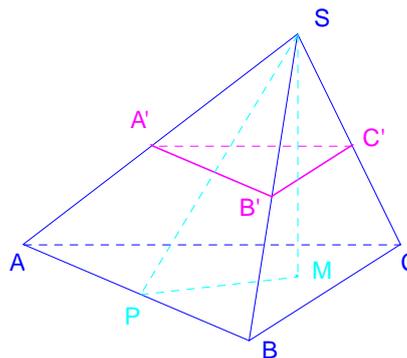
How to construct the template of such a pyramid ?
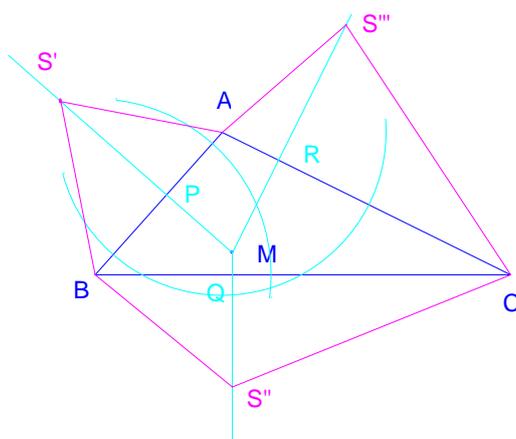
The height (SP) of the lateral face SAB remains
perpendicular to the side (AB) when the face is turned
down around (AB), thus (MS') is perpendicular to (AB),
(MS") to (BC) and (MS''') to (CA).

Construct the three perpendicular lines to ABC's sides
going through M. Choose S' on one of them (P between
S' and M, with PS'>PM) ; S" and S''' are on (MQ) and (MR)
so as AS'''=AS' and BS"=BS'.

You may choose the height h of the pyramid.
Just construct a right triangle with his perpendicular
sides equal to MP and h ; the hypotenuse is PS'.

The construction works with M outside of ABC.
And *with a quadrilateral base ?* It works, even if the base is non convex, as it does
with any non-convex polygon with non-crossed sides.

And what about A'B'C' ? It is the section of the pyramid SABC by a plane parallel to
ABC ; we have reduced the pyramid !
What is the ratio of the volumes of the two pyramids ? k^3 of course (in SAM the
theorem proves that the ratio of the heights is k).
These results may be generalized to any convex polyhedron : just divide it into
pyramids (chose a point inside and join it to all the vertices).

Move the tetrahedron, and you'll find a view where the heights of the three
trapeziums ABB'A', BCC'B' and CAA'C' appear to be equal.
What is the direction of that view ? the direction of (SI) where I is the centre of the
circle inscribed in ABC.

" **Geometry is a skill of the eyes and the hands as well as of the mind.** " (Jean Pedersen)

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http://www.ac-noumea.nc/maths/amc/polyhedr/
"a riding through the polyhedra's world"
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