**Polyhedron Man**

# Spreading the word about the wonders of crystal-like geometric forms

**Ivars Peterson**

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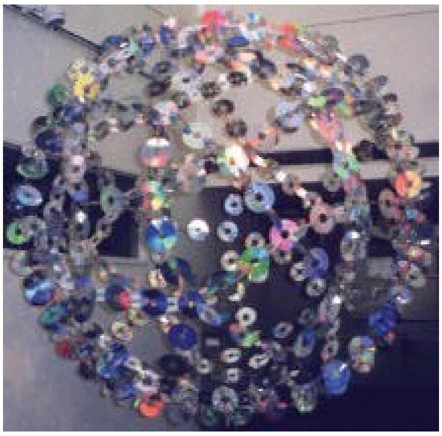

*George Hart's "Rainbow Bits".*
Séquin

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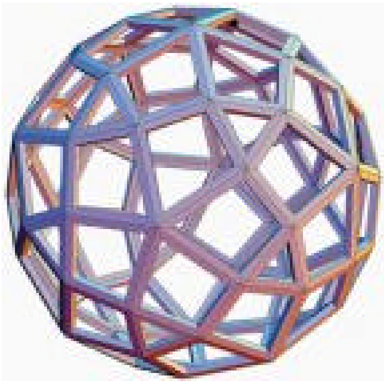
Created by George W. Hart of Northport, N.Y., the sculpture is more than an intriguing visual effect. Its strands of recycled CDs trace the edges of a novel mathematical solid—a polyhedron made up of 20 equilateral triangles and 60 kite-shaped quadrilaterals.



*Computer-generated illustration of the geometric solid—a propellor icosahedron—that served as the basis for the sculpture shown above.*
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This particular solid, which Hart dubs a propellorized icosahedron, is one example of a newly defined class of polyhedra. "As far as I know, these forms have never previously been described by mathematicians," Hart says.

Hart's Berkeley sculpture is just one of his many creations based on polyhedra. Hart has also collaborated with mathematicians and computer scientists to define and visualize a variety of new mathematical shapes.

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Such forms tantalize artists and scientists alike. "George Hart is a very special individual, a gifted mathematician, a creative artist, and an inspiring teacher," says Berkeley computer scientist Carlo H. Séquin. "He is among the very best of a rather small group of people who successfully integrate art and mathematics. Hart's geometrical constructions are intriguing, educational, and artistic at the same time."


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**Passion for polyhedra**

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Hart majored in mathematics, then earned graduate degrees in linguistics and in electrical engineering and computer science. He taught computer and engineering courses at Columbia University and Hofstra University in Hempstead, N.Y. Along the way, he wrote a textbook on multidimensional analysis, a branch of mathematics that relates physical measurements to one another. He also developed and patented methods for determining the energy consumption of individual appliances from measurements of total power used by a household.

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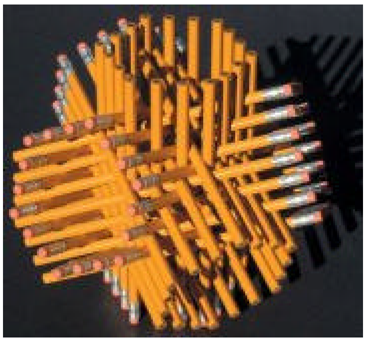
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**Linking polygons**

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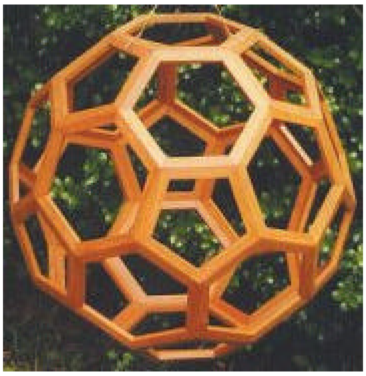

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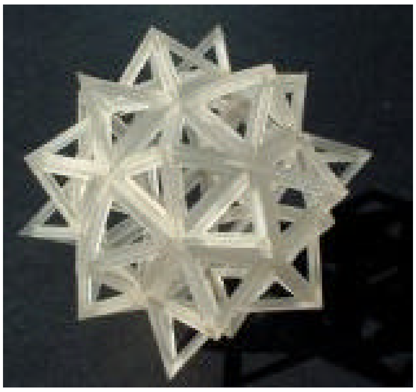

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**Generating novel forms**

Working with Craig S. Kaplan of the University of Washington in Seattle, Hart has recently invented a procedure for generating many polyhedral forms, including new types of polyhedra. Dubbed symmetrohedra, these geometric solids result from the symmetric placement of regular polygons (see illustration below).

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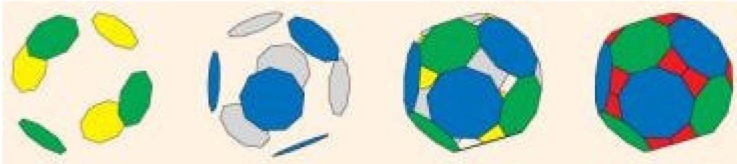

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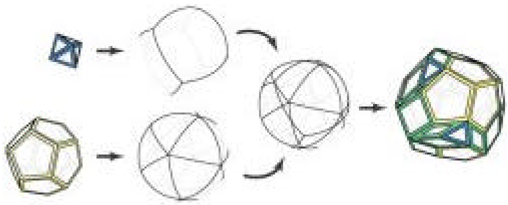

*Blending an octahedron (top left) and a dodecahedron (bottom left) produces another sort of polyhedron (right).*
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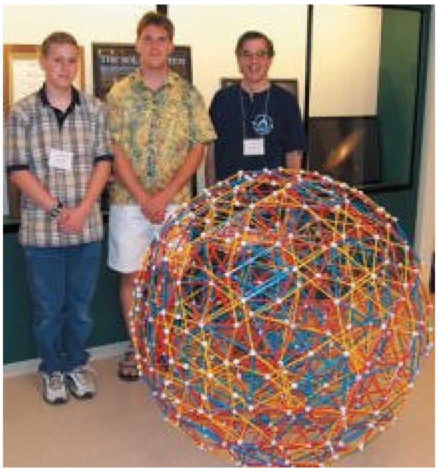
**Beauty and joy**

Whether assembling an intricate polyhedral design, leading a model-building workshop, or describing his latest ventures, Hart brings an infectious enthusiasm to his task. To help spread the word about the beauty and joy of polyhedra, he recently coauthored a book on building models using Zome plastic components, produced by Zometool of Denver. Embodying mathematical ratios such as the golden mean, the system's balls and sticks lend themselves to polyhedron building. On another communications front, Hart is now writing a history of polyhedral geometry in art.



*George Hart (right) with high-school student Thomas Engdahl (left), David Richter (middle) of Southeast Missouri State University, and an elaborate model constructed from Zome components.*
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Zongker, D., and G.W. Hart. 2001. Blending polyhedra with overlays. In *Bridges: Mathematical Connections in Art, Music, and Science Conference Proceedings*,>, R. Sarhangi and S. Jablan, eds. See http://www.sckans.edu/~bridges/.

***Further Readings:***

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______. 1995. *Multdimensional Analysis: Algebras and Systems for Science and Engineering*. New York: Springer-Verlag. See http://www.georgehart.com/research/multanal.html.

Images of some of George Hart's geometric sculptures are available at http://www.georgehart.com/sculpture/sculpture.html.

Learn more about George Hart's "Rainbow Bits" at http://www.cs.berkeley.edu/~sequin/SCULPTS/RAINBOW/index.html and http://www.georgehart.com/sculpture/rainbow-bits.html.

Information about the "Bridges: Mathematical Connections in Art, Music, and Science" conference can be found at http://www.sckans.edu/~bridges/.

Information about the Conway notation for polyhedra can be found at http://www.georgehart.com/virtual-polyhedra/conway_notation.html.

Zometool has a Web site http://www.zometool.com/.

Additional information about the Zome model of a rectified 600-cell, constructed by David Richter and his coworkers, can be found at http://cstl-cst.semo.edu/richter/bridgeszome.htm.

***Sources:***

Bob Brill
Web site: http://users.migate.net/~bobbrill/

George W. Hart
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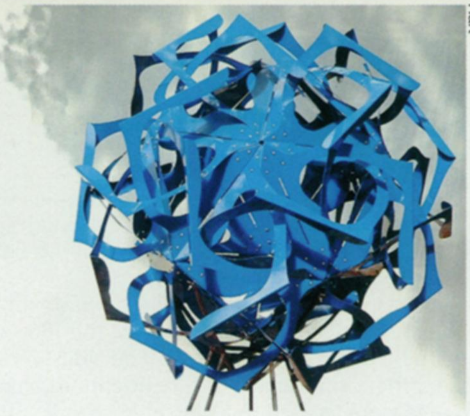

George Hart's "Twisted Rivers."

By IVARS PETERSON

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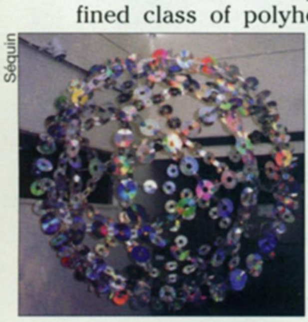

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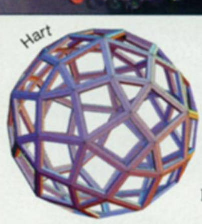

George Hart's "Rainbow Bits". (top) Computer-generated illustration of the geometric solid—a propellor icosahedron (bottom)—that served as the basis for the sculpture.

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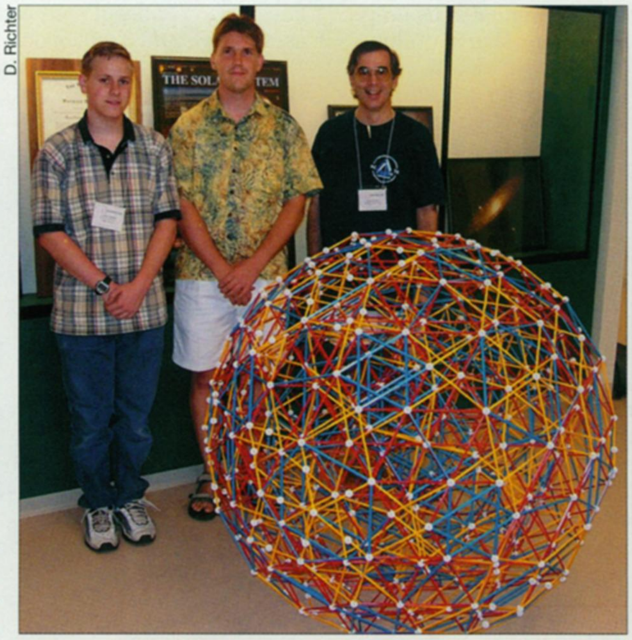

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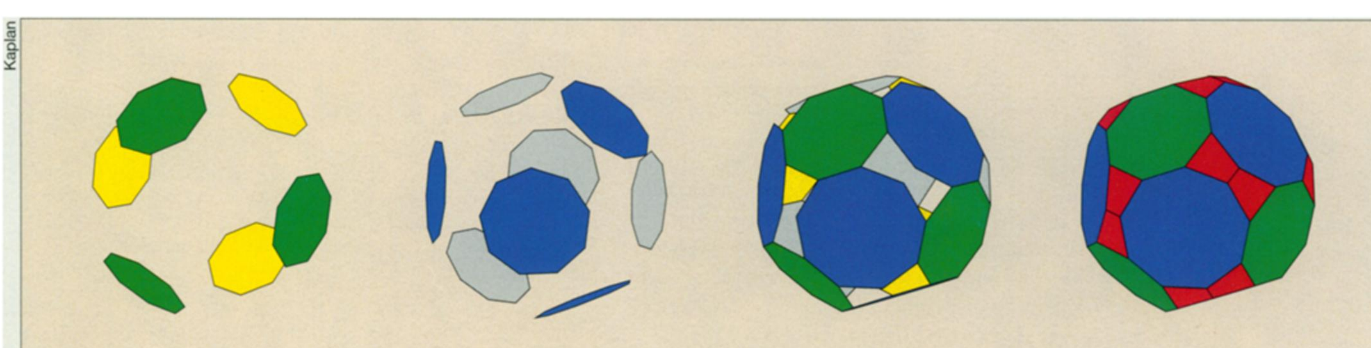

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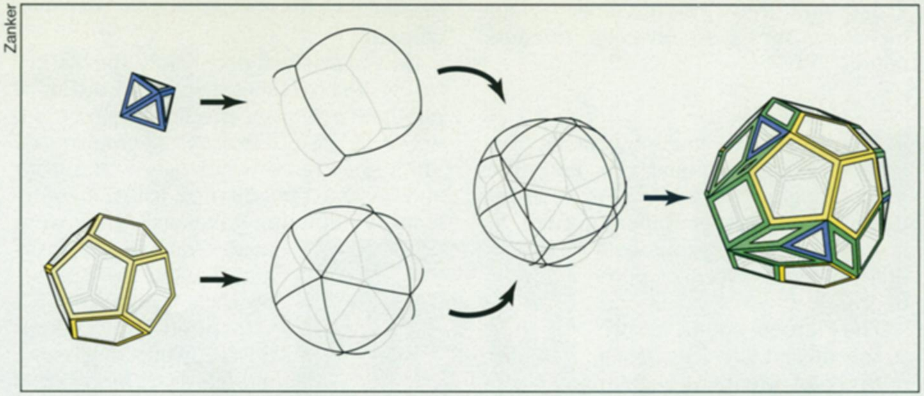

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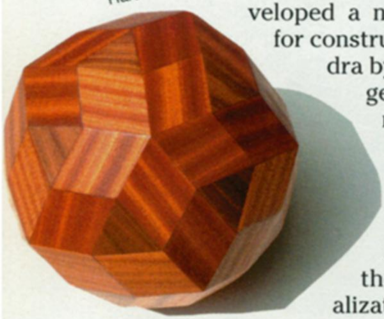

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